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LETTER TO THE EDITOR

Analogue experiments and computer simulations for directed conductivity

S Redner† and J S Brooks‡

†Center for Polymer Studies§ and Department of Physics, Boston University, Boston MA 02215, USA

‡ Department of Physics, Boston University, Boston MA 02215, USA

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Abstract. We study the conductivity in directed percolation by an analogue experiment and by computer simulations. In the experiment, the dependence of the conductivity on concentration of conducting diodes, p, shows very large fluctuations. This reflects the topological constraints of directed percolation where conduction is very sensitive to singly connected or cutting bonds. We have checked this hypothesis by studying the underlying directed backbone of the experiment using computer simulations. From the dependence of the conductivity on the backbone fraction, and the dependence of the backbone fraction on $p - p_c$, we infer a directed-conductivity exponent considerably less than unity, consistent with earlier theoretical work.

In this letter, we study the conductivity in directed percolation both by an analogue experiment and by computer simulations. We are motivated in part by the recent theoretical work on directed conductivity (Redner 1982b, Redner and Mueller 1982, Dhar *et al* 1982). The theoretical work predicts that the directed conductivity should vanish as $(p - p_c)^{t_+}$, as p, the concentration of conducting bonds, approaches a critical concentration p_c from above. The numerical value for the directed conductivity exponent, t_+, was found to be approximately $\frac{2}{3}$, rather different from recent estimates of close to $\frac{4}{3}$ for the conductivity exponent of the isotropic random resistor network (see e.g. Fogelholm 1980, Lobb and Frank 1982, and references therein). Our study provides evidence in support of this small value of t_+ by an analysis of our conductivity experiment in terms of the underlying directed backbone of the network.

A typical network considered in this work is sketched in figure 1. Each bond is a resistor and diode in series so that its I–V characteristic has the form shown in figure 2. In our computer simulations, we have approximated this response by a simpler model in which a bond behaves like an ideal resistor if it is forward-biased, and like an open circuit if it is back-biased. As we shall see, this modification does not change the conductivity of the network significantly for the applied voltages at which the experiment was performed.

A second feature of the networks is that they are rotated by 45° with respect to the applied field. Our reason for doing this is that directed percolating systems are anisotropic in structure near the percolation threshold. This anisotropy picks out two special axes, one parallel and one perpendicular to the direction of the applied field as shown in figure 1 (see e.g. Kinzel and Yeomans 1981, Klein and Kinzel 1981,

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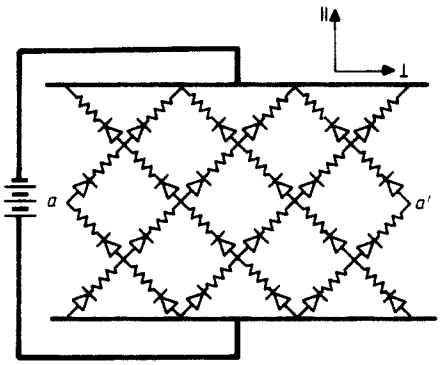

Figure 1. Schematic picture of the directed percolating system considered in this work. The parallel and perpendicular axes of the network are indicated. In the experiments, both free and periodic boundary conditions were employed in the transverse direction. For periodic boundaries, the points a and a' are identical.

Redner 1982a, and references therein). Along these two directions, distinct correlation lengths diverge at different rates. Because of this anisotropy, the conductivity is a tensor. By orienting the lattice by 45°. the conductivity tensor becomes diagonal, and our measurements examine only its largest component.

Experimentally, we have studied networks of sizes 10×10, 19×19 and 19×52 sites. The largest lattice was chosen to be rectangular in shape as dictated by anisotropic finite-size scaling considerations for directed percolation (Redner and Mueller 1982). To perform the experiment, the coordinates of each lattice diode were generated and

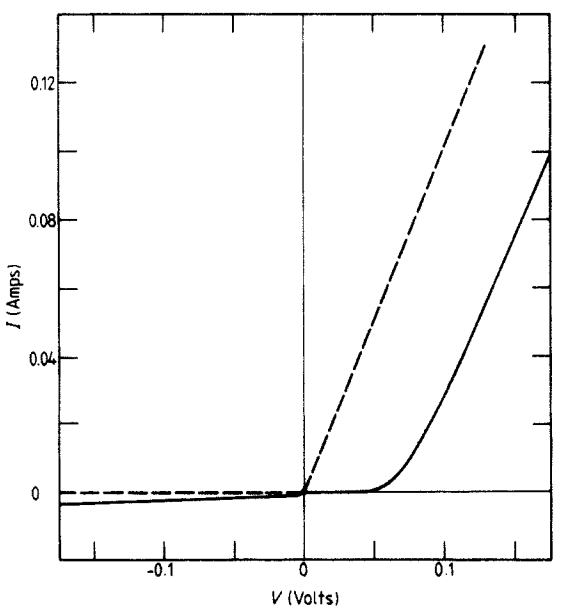

Figure 2. Experimental current–voltage characteristic of a single bond consisting of a resistor and diode in series. We also show (broken line) the I–V characteristic of an ideal 'ohmic' diode on which the computer simulation is based.

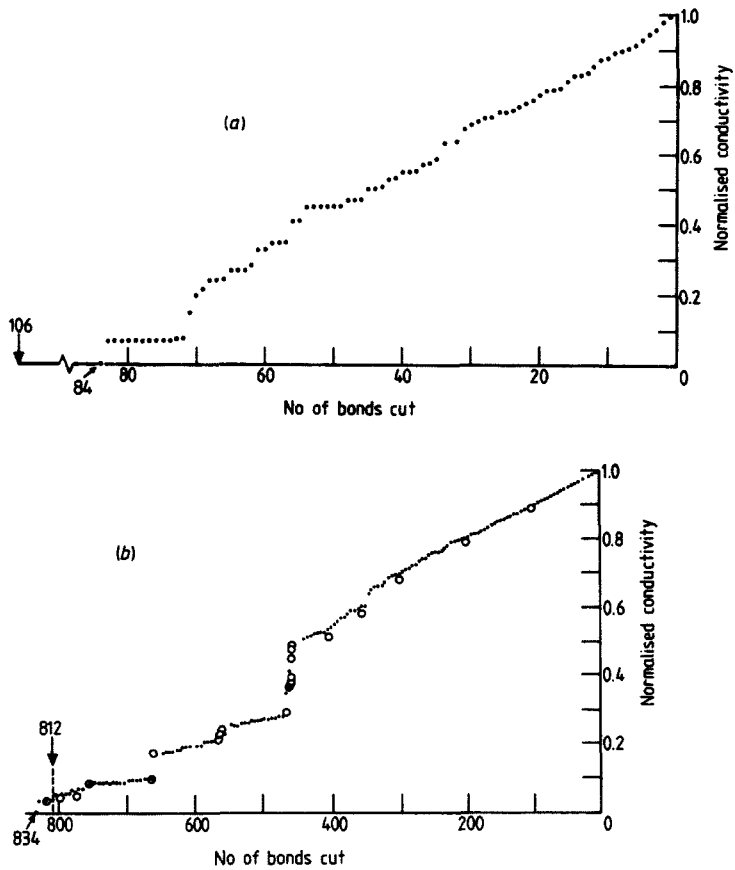

Figure 3. (*a*) Normalised conductivity as a function of concentration p for the 10×10 lattice. The values of p_c predicted by computer simulations and from the analogue experiment are indicated by the arrows. (*b*) Same as (*a*), but for the 19×52 lattice. The open circles are the results of the simulations which are concentrated in the vicinity of the conductivity jumps.

stored in a random order,and the diodes were then cut sequentially according to the ordering of this list. For the smallest lattice (figure 3(*a*)), the data for the conductivity as a function of p is quite linear for a large range of p, as would be predicted by an effective-medium theory. Close to p_c, the conductivity shows downward curvature, qualitatively consistent with a directed conductivity exponent of $t_+ \lesssim 1$.

When we look at the 19×52 lattice (figure 3(*b*)), the situation is more interesting. As a function of concentration, the conductivity shows several rather large jumps, one being approximately 22% of the normalised conductivity. These fluctuations are several orders of magnitude larger than those encountered in similar analogue experiments for a random resistor network (see e.g. Watson and Leath 1974). Identical jumps occur when our experiment is performed with either 60V or 150V across the network. This indicates that the finite voltage threshold (figure 2) for turning on each diode plays an insignificant role in the conductivity, and that the jumps arise from purely geometrical effects. We have therefore employed computer simulations in order to understand the origin of the jumps and to serve as a guide for interpreting the conductivity data.

For the computer simulations, we have used the stored list of randomly ordered bonds to repeat the analogue experiment, focusing primarily on the regions of the large jumps (figure 3(*b*)). The network conductivity was calculated by a numerical relaxation method as outlined in Redner and Mueller (1982). In addition, we examined the geometry of the network directly. Figure 4(*a*) shows the backbone of the network after 465 bonds have been removed and all isolated clusters and dangling ends have been stripped away. In the figure, the next bond to be removed in the experiment is indicated by the arrow. Notice that in isotropic percolation, the removal of this arrowed bond would entail the stripping away of only four additional dangling bonds in order to obtain the new backbone. However in directed percolation, 140 bonds must be stripped in order to obtain the backbone (figure 4(*b*)). The normalised directed conductivity of the network is 0.371 before the arrowed bond is removed, and 0.294 afterwards. In sum, the simulations confirm that the jumps in the conductivity arise because of large changes in the size of the backbone.

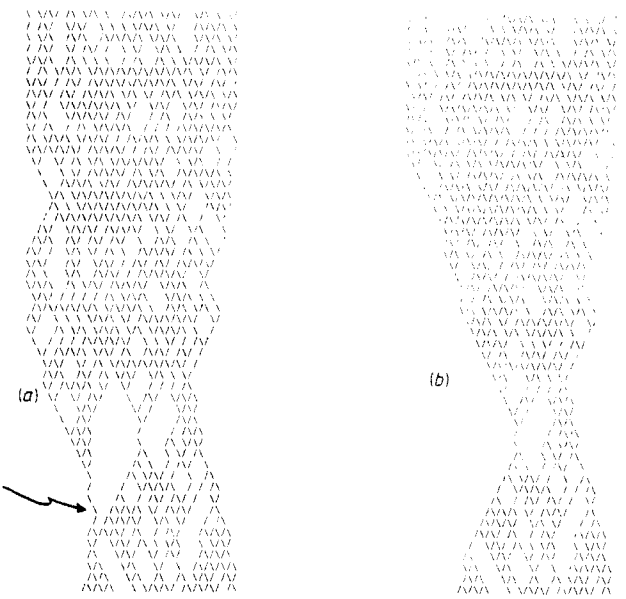

Figure 4. (*a*) The directed backbone of the 19×52 network after 465 bonds have been removed. The arrow indicates the next bond that will be removed. (*b*) The directed backbone after the arrowed bond in (*a*) is removed.

The strong correlation between the conductivity and the backbone size suggests that we first examine the dependence of the conductivity on the number of bonds in the backbone, and then determine the backbone size as a function of $(p - p_c)$ independently. From the 19×52 experiment, we plot the conductivity versus backbone fraction in figure 5(*a*). This data is much better behaved than the dependence of the conductivity on p, and it appears that the conductivity varies approximately linearly with the backbone fraction.

In order to study the behaviour of the backbone fraction on $p - p_c$, we use anisotropic finite-size scaling which predicts that for a finite system at $p = p_c$, the backbone fraction should vary as $L_\parallel^{-\beta'/\nu_\parallel}$. Here β' is the directed backbone exponent, $\nu_\parallel \cong 1.74$ is the parallel correlation length exponent of directed percolation, and $L_\parallel$ is

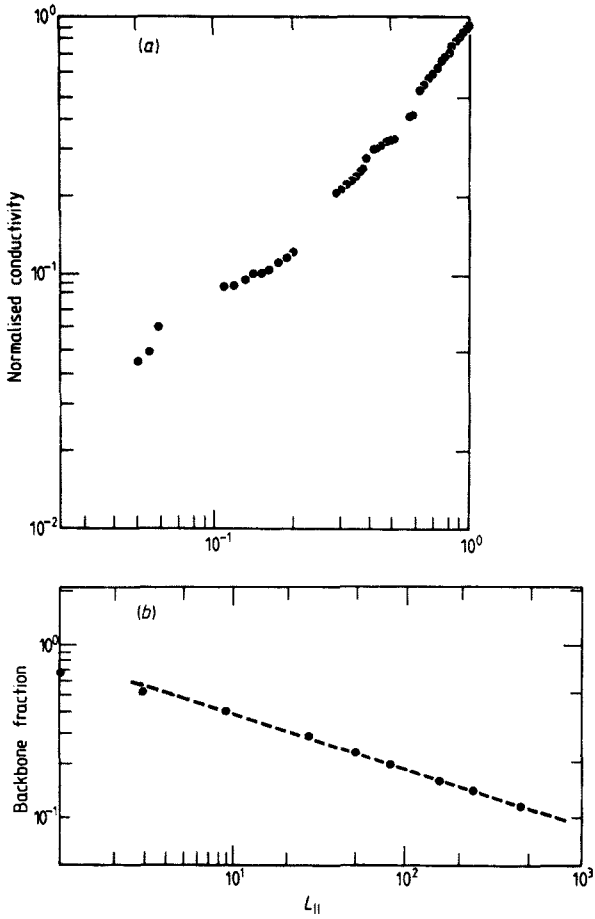

Figure 5. (*a*) Plot of the conductivity against backbone fraction, and (*b*) the backbone fraction against the network length, $L_{\|}$. The latter data are based on anisotropically scaling a 1×1 lattice up to a 48×453 lattice. In (*b*) the broken line has a slope of -0.32.

the length of the system. On a double logarithmic scale, a plot of backbone fraction versus $L_{\|}$ has an asymptotic slope of -0.32 (figure 5(*b*)), or an exponent $\beta' = 0.56$.

In summary, we have found that the large jumps observed in the conductivity of directed percolation arise from fluctuations in the backbone fraction. A much smoother variation results if the conductivity is plotted against the backbone fraction instead of the total number of bonds. Since the directed backbone exponent and the directed conductivity exponent appear to be fairly close, we therefore conclude that the latter exponent is considerably less than unity. This is in qualitative agreement with previous analytical and numerical work on this problem.

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Note added. As we were writing up this work, we learned that Dhar *et al* have also studied the directed conductivity problem. They obtain results similar to ours, and

an additional result that $\beta' = 2\beta$, where β is the exponent of the directed percolation probability. Our estimate for β' is in excellent agreement with this latter result. We are grateful to D Dhar for communicating his results prior to publication.

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